Integration (Mathematics 1& 2, quide)

February 29, 2020

4.4 Integration by substitution

E 4.1 $\int x(3x+5)^7\,dx$

E 4.2 $\int x(2x^2+7)^8\,dx$

E 4.3 $\int x^3(2x^2+7)^8\,dx$

E 4.4 $\int \frac{x+1}{x^2+2x+7}\,dx$

E 4.5 $\int \frac{x}{x^2+2x+7}\,dx$

4.4 Integration by substitution

A 4.5 $\int xe^{x^2}\,dx$

A 4.6 $\int x\sqrt{2x^2+2}\,dx$

A 4.7 $\int x\sqrt{x-1}\,dx$

E 4.9 $\int_0^1 (3x+5)^2\,dx$

A 4.8 $\int_0^1 \frac{x+2}{x^2+4x+5}\,dx$

4.5 Integration by parts

E 4.10 $\int x \ln x \, dx$

E 4.11 Why the integral of basic function $\ln x$ isn't on basic list?

$$\int \ln x \, dx$$

A 4.9 $\int x e^x \, dx$

4.6 Partial fractions

E 4.12 $\displaystyle\int \frac{x}{x^2 - x - 2}\, dx$

A 4.10 $\displaystyle\int \frac{dx}{x^2 + 4x + 3}$

4.7 Applications of integration

E 4.13 Suppose that the marginal cost function is given $MC(q) = 2e^{0.5q}$ and that the fixed cost is 20. Determine the total cost function TC.

A 4.11 Find the total cost function if the marginal cost is $q + 5q^2 + e^q$ and the fixed cost is 10

Sample examination / practice questions

Q4.1 Determine the following integral:

$$\int \frac{x}{x^2+5x+6}dx$$

Q4.2 Evaluate $\int_1^2 x^2(x-1)^{1/2}dx$ using appropriate substitution.

Q4.3 Determine $\int \frac{2x+3}{x^2+3x+2}dx$

Q4.4 Determine $\int_1^e \frac{\ln x\sqrt{\ln x}}{x}dx$

Sample examination / practice questions

Q4.5 A company produces only product XYZ. When producing Q units the marginal cost MC is given by

$$MC = 1 - \frac{1}{(Q+1)^2}$$

If average cost per unit when producing 4 units is 3.05, what is total cost of producing 5 units of XYZ?

Sample examination / practice questions

Q4.6 A company's marginal cost function is

$$MC = 32 + 18q - 12q^2$$

It's fixed cost is 43. Determine the firm's total cost function, average cost function, and variable cost.

Sample examination / practice questions

Q4.7 The marginal revenue function for o commodity is given by

$$MR = 10 - 2x^2,$$

and the total cost function for a commodity is

$$TC = x^2 + 4x + 2,$$

where x is the number of units produced. Find the revenue function, and determine the maximal profit.

Sample examination / practice questions

Q4.8 For a particular company, the marginal cost is a function of output as follows:

$$MC = 10 - q + q^2$$

Determine the extra cost which is incurred when production is increased from 2 to 4.

Sample examination / practice questions

Q4.9 A firm's marginal cost function is

$$\frac{20}{\sqrt{Q}}e^{\sqrt{Q}} + Q^3 + \frac{1}{Q+1}$$

The firm's fixed costs are 20. Determine the total cost function.

Appendix A - Sample examination paper

[1.(2009-2010)]
A firm is only producer of a particular good, and the demand equation for a good is $2p + q = 20$, where p denotes the selling price and the q is quantity produced by the firm. The firm's fixed costs are 12 and the marginal cost function is $MC = 2 + q$. Find an expression for the profit function, $\Pi(q)$. Sketch the graph of Π for q between o and 8. Determine the value of production, q, which maximizes the firm's profit.

Appendix A - Sample examination paper

[3.(2009-2010)]
Determine integrals

$$\int x(x+2)^{5/2}dx \qquad \text{and} \qquad \int \frac{e^x}{e^{2x}-1}dx.$$

Quide 2, Sample examination paper

[6.(2009-2010)]
The supply equation for a good is $p = 3 + 3q + 2q^2$ where q denotes the quantity supplied and the p the price. Determine the equilibrium quantity and the producer surplus.
If a elasticity of demand for the good is equal to 1 for every value of the price, determine the demand function.