Techiniques of integration

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Mathematics for economics and finance

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Integration by substitution

- **Example:** Find indefinite integral:

$$\int (7x+2)^{11} dx$$

- How dx is related to dt
- General rule

$$\int f(x)dx = \int f(x(t))x'(t)dt$$

- Finding a good substitution !
- **Example:** Find the integral:

$$\int x\sqrt{3x^2+5}\,dx$$

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Definite integrals by substitution

- General rule

$$\int_{x=a}^{x=b} f(x)dx = \int_{t=\alpha}^{t=\beta} f(x(t))x'(t)dt,$$

  where $x(\alpha) = a$ and $x(\beta) = b$

- **Example:** Evaluate integral:

$$\int_{x=2}^{x=3} x\sqrt{3x^2+5}\,dx$$

- **Example:** Calculate

$$\int_{x=0}^{x=5} \frac{x+1}{x^2+2x+5}\,dx$$

Integration by parts

- General rule

$$\int u'(t)v(t)dt = u(t)v(t) - \int u(t)v'(t)dt$$

  prove, shorthand
- Finding a good parts, ideas !
- **Example:** Find the integral:

$$\int t \ln t \, dt$$

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Partial fractions

Based on algebraic method for rewriting particular expression in a simpler form.

- **Example:** Find the integral:

$$\int \frac{t+1}{t^2+t-2}\,dt$$

- General rule

$$\frac{p(t)}{q(t)} = \frac{p(t)}{C(t-a_1)(t-a_2)\cdots(t-a_m)} = \frac{A_1}{t-a_1} + \frac{A_2}{t-a_2} + \cdots \frac{A_m}{t-a_m}$$

- **Example:** Find the integral:

$$\int \frac{2x+1}{x^2+5x+6}\,dx$$

Worked examples 26.1

Find the integrals

$$\int \frac{2t+1}{t^2+t+1}\,dt, \quad \text{and} \quad \int_0^2 \frac{2t+1}{t^2+t+1}\,dt$$

Worked examples 26.2

Find an anti-derivative of $1/(t^2 - 1)$ and hence calculate the definite integral

$$\int_2^7 \frac{1}{t^2 - 1}\, dt$$

Worked examples 26.3

Evaluate

$$\int_0^1 t^2 e^t \, dt$$

Worked examples 26.4

Find the area under the curve $f(t) = t^2 e^{t^3}$ between the lines $t = 1$ and $t = 3$.

Worked examples 26.5

Calculate

$$\int_1^2 \frac{\sin(\ln x)}{x}\, dx$$

Worked examples 26.6

Show that if $f(x)$ and $f'(x)$ are both positive in the interval $[a, b]$, then

$$\int_a^b \frac{f'(x)}{f(x)}\,dx = \ln \frac{f(a)}{f(b)}$$