Integration (Mathematics 2, exam)

LSE

February 29, 2020

Preliminary exam 2015

2. A market has an inverse supply function and an inverse demand function given by

$$p^S(q) = 1 + 2q \qquad \text{and} \qquad p^D(q) = 6 - 3q,$$

respectively. Find the equilibrium price and quantity of the market.
A percentage of the price tax of $100r\%$ is imposed on the market. What is the new equilibrium price and quantity for the market? Find the consumer surplus in the presence of this tax, writing your answer in terms of r and simplifying your answer as far as possible.

Exam 2015, ZA

2. For positive constants k, a and b, the supply and demand functions for a particular good are given by

$$q^S(p) = p - b \qquad \text{and} \qquad q^D(p) = \frac{k}{p} - a,$$

respectively. Given that equilibrium quantity is $a + b$, find k in terms of a and b. Hence find an expression, in terms of a and b, for the consumer surplus

Exam 2016, ZA

1. In a market, the inverse supply function is given by $p^S(q) = aq + b$ for some positive constants a and b, the equilibrium price is 7 and the equilibrium quantity is 2. Given that the producer surplus is 4, find a and b.

Exam 2016, ZB

1. In a market, the inverse demand function is given by $p^D(q) = a - bq$ for some positive constants a and b, the equilibrium price is 4 and the equilibrium quantity is 1. Given that the consumer surplus is 1, find a and b.

Exam 2016, MOCK

1. At what points do the curves $y = x^2$ and $y = 2 - x$ intersect? Sketch these two curves on the same set of axes.
   On your sketch, indicate the region which is under both of these curves, above the x-axis and between the vertical lines $x = 0$ and $x = 2$. Use integration to find the area of this region.

7. a) In a market with demand equation $q = q^D(p)$, the elasticity of demand is given by

   $$\varepsilon(p) = -\frac{p}{q}\frac{dq}{dp},$$

   and the revenue generated by the market is $R(p) = pq = pq^D(p)$. Show that, if the elasticity of demand is greater than one, then $R(p)$ is a decreasing function of p.

   b) Given that a market's demand equation is

   $$q^2(3 + p^3) = 44;$$

   find the elasticity of demand and show that it never exceeds $3/2$.
   For what values of p is the revenue from this market a decreasing function of p?

   c) The inverse supply function for the market in (b) has the form $p^S(q) = \alpha q + \beta$ where α and β are positive constants.
   Given that the equilibrium point for this market is $(q, p) = (2, 2)$ and the producer surplus is one, find the values of α and β.

Exam 2017, Mock

1. The supply equation for a market is given by $q = p - 4$ and the demand equation is $p(q + 2) = 24$ where p is the price and q is the quantity. Find the equilibrium price and quantity for this market.
   Show that, in this market, the consumer surplus is $12(2\ln(2) - 1)$ and find the producer surplus.

Exam 2018, ZA

1. The inverse supply and demand function for a commodity are given by

$$p^S(q) = q + 45 \qquad \text{and} \qquad p^D(q) = \frac{500}{q + 5},$$

respectively. Find the equlibrium price and quantity of the market. Hence find the consumer and producer surplus.

Exam 2018, ZB

1. The inverse supply and demand function for a commodity are given by

$$p^S(q) = 2q + 11 \qquad \text{and} \qquad p^D(q) = \frac{231}{2q + 11},$$

respectively. Find the equlibrium price and quantity of the market. Hence find the consumer and producer surplus.

Exam 2018, Mock

1. In a market for a good, the price p and quantity q are related by the supply equation $q = p + 1$. Given that the equilibrium price is 1, find the equilibrium quantity. If the elasticity of demand for this good is $\dfrac{p^2}{p^2 + 3p + 2}$ find the demand function, $q^D(p)$.