Integration (Mathematics 1, exam)

LSE

February 29, 2020

Preliminary exam 2014

5. Determine the following integrals

   a) $\displaystyle\int \frac{\ln(x+1)}{x^2}\,dx$

   b) $\displaystyle\int \frac{\cos x}{(1-\sin x)(2+\sin x)}\,dx$

   c) $\displaystyle\int e^{\sqrt{x}}\,dx$

Exam 2015, A

1. A monopolist's marginal cost function is given by

$$MC = 20 + 4q,$$

where q is quantity of good produced. Her fixed costs are 20, and the demand equation for good she produces is

$$p + 4q = 40,$$

where p and q are price and quantity, respectively. Find the expressions for the total revenue and for the profit, as function of q. Determine the value of q which maximisees the profit.

7. (b) Determine the following integrals

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$$

$$\int_1^e x^2 \ln x\,dx$$

Exam 2015, B

1. A monopolist's marginal cost function is given by

$$MC = 10 + 2q,$$

where q is quantity of good produced. Her fixed costs are 20, and the demand equation for good she produces is

$$p + 2q = 20,$$

where p and q are price and quantity, respectively. Find the expressions for the total revenue and for the profit, as function of q. Determine the value of q which maximisees the profit.

7. (b) Determine the following integrals

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$$

$$\int_1^e x^3 \ln x\,dx$$

Exam 2015

2. The function f is defined for $x > 0$ by

$$f(x) = ax^2 + bx + \frac{c}{x},$$

for some constants a, b, c. If $f(1) = -5$, $f'((1) = -1$ and $\int_1^2 f(x)dx = \ln 2 - 4$, show that the following system of equations holds for a, b and c:

$$a + b + c = -5, \quad 2a + b - c = -1, \quad 14a + 9b + (6\ln 2)c = 6\ln 2 - 24$$

Express this system of equations in matrix form.
Write down the augmented matrix for this system of equations and use row operations to determine the values of a, b and c

5. Determine the following integrals:

a) $\int x^3 \ln(x^2 - 1)\, dx$

b) $\int \frac{x}{\sqrt{e^x}}\, dx$

c) $\int \frac{x}{x^2 + 5x + 4}\, dx$

Exam 2016, A

1. A monopoly has fixed cost of 20 and marginal cost function $3q^2 + 4$. The demand equation for its product is $p + 4q = 40$.
   Determine the profit function in terms of q.
   Hence find the production level that maximises the profit.

4. Determine the following integrals

$$\int \frac{x+1}{x^2+7x+10}\,dx$$

$$\int \frac{\ln(2x+1)}{\sqrt{2x+1}}\,dx$$

Exam 2016, B

1. A monopoly has fixed cost of 10 and marginal cost function $6q^2 + 8$. The demand equation for its product is $p + 2q = 40$.
   Determine the profit function in terms of q.
   Hence find the production level that maximises the profit.
4. Determine the following integrals

$$\int \frac{x}{x^2 + 7x + 12}\, dx$$

$$\int \frac{\ln(x+1)}{\sqrt{x+1}}\, dx$$

Exam 2016

1. A firm is only producer of a particular good. The firm's marginal revenue function is $MR = 9 - q$, where q denotes the quantity of the good produced by rhe firm. The firm's fixed cost's are 12 and it's average variable cost function is $1 + q/2$. Find an expression for the firm's profit function. $\Pi(q)$. Find the value of the production, q. which maximises the firm's profit, and hence calculate the firm's maximun profit.

5. (a) Determine the integral

$$\int \frac{x^{-1}dx}{(1+\ln x)\ln x}$$

7. (b) A firms cost function is given by

$$MC = qe^q + \frac{q}{q^2+4q+3},$$

where q denotes the quantity produced. If its production is increased from $q = 1$ to $q = 2$, use integral to find the increase in total cost.

Exam 2017, A

2. Determine the following integrals

$$\int x^2 e^x \, dx$$

$$\int x^2 \sqrt{x-2} \, dx$$

7. (b) Determine integral $\int \frac{1}{1+e^{2x}} \, dx$

Exam 2017, B

2. Determine the following integrals

$$\int x^2 e^x \, dx$$

$$\int x^2 \sqrt{x-1} \, dx$$

7. (b) Determine integral $\int \frac{1}{1+e^{2x}} \, dx$