Areas and integrals

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Mathematics for economics and finance

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The consumer surplus - 1

- The consumer surplus
- $CS = A - R$ - Measure of the benefit consumers derive from the operation of the market

The Consumer surplus - Example 1.

Suppose that the demand set for a given good is

$$D = \{(q, p) | q + 200p = 2600\},$$

and the supply is

$$S = \{(q, p) | q - 100p = -1000\}.$$

- Find the equilibrium point on that market
- Find the consumer surplus (CS)

The concept of area and anti-derivatives (integrals)

- Suppose that the function f is such that $f(x) \geq 0$ for all x in some interval $[0, X]$.
- Derivative of the function A, which measures the area under the graph of f, is just given function f
- If we are given the function f, the area A may be obtaining by finding a function which, when differentiated, gives f. $F'(x) = f(x)$
- Let $\int f(t)dt = F(t) + c$. Primitive function, constant of integration, integral, integrand
- General form of anti-derivative of f is called *indefinite* integral of $f(t)$, and denoted

$$\int f(t)dt.$$

Anti-derivatives and integrals

- Find anti-derivative of $f(t) = t^2$
- If the functions F and G are both anti-derivative of f. What is the difference between this two functions?
- Prove that for all $\alpha \neq 1$

$$\int t^{\alpha} dt = \frac{1}{\alpha + 1} t^{\alpha+1} + c$$

- Two basic properties of integrals
- Find $\int (5t^2 + 7t + 3) dt$

Definite integrals

- Let f be a function with an anti-derivative F. The *definite integral* of the function f over the interval $[a, b]$ is:

$$\int_a^b f(t)dt = F(b) - F(a).$$

- Does the quantity on the right-hand side of the definition depend on which of anti-derivatives is chosen?
- Shorthand for right side
- Example: Find the definite integral of t^2 over $[1, 2]$
- Relationship between the definite integral and the area under curve? Basic / extended
- Example: What is the area under curve $y = t^3$ between $t = 2$ and $t = 4$.

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Standard integrals

- Table of 5 standard integrals.
- Find the consumer surplus from Example 1. using definite integrals
- **Example 2.** Suppose that the demand set for tins of caviar is

$$D = \{(q, p) | p^3 q = 8000\},$$

and the supply is

$$S = \{(q, p) | q = 500p\}.$$

  - Find the equlibrium point on that market
  - Find the consumer surplus (CS)

Worked examples - Example 25.1

Find the area enclosed by the lines $t = 1$, $t = 2$, the t-axis, and the graph of the function $f(t) = e^t$.

Worked examples - Example 25.2

Show that when $t > -1$ the derivative of $\ln(t+1)$ is $\frac{1}{t+1}$. Show also that

$$\frac{t^2}{t+1} = t - 1 + \frac{1}{t+1},$$

and hence find the indefinite integral

$$\int \frac{t^2}{t+1} dt.$$

Evaluate the definite integral

$$\int_2^3 \frac{t^2}{t+1} dt.$$

Worked examples - Example 25.3

Suppose that the demand set and supply set for good are

$$D = \{(q, p) | q = c - dp\}, \qquad S = \{(q, p) | q = bp - a\},$$

where a, b, c, d are positive constants. Find the expression for the consumer surplus

Worked examples - Example 25.4

Find the area enclosed by the curves $y = 1/t^2$, $y = t^3$, the t-axis, and the lines $t = 1/2$ and $t = 2$

Worked examples - Example 25.5

The demand set for a commodity is

$$D = \{(q, p) | p(q + 1) = 231\},$$

and the supply is

$$S = \{(q, p) | p - q = 11\}.$$

Determine the consumer surplus.