Exercise 25: Integrals

1. Show that the derivative of $\ln(t+3)$ is $\frac{1}{t+3}$. Show also that

$$\frac{t^2+4t+4}{t+3} = t+1+\frac{1}{t+3},$$

and hence find the indefinite integral

$$\int \frac{t^2+4t+4}{t+3}dt.$$

Evaluate the definite integral

$$\int_1^2 \frac{t^2+4t+4}{t+3}dt.$$

2. The demand set for a commodity is

$$D = \{(q,p)|q = 170 - p\},$$

and the supply is

$$S = \{(q,p)|2q + 140 = p\}.$$

Determine the consumer surplus.

3. Find the area enclosed by the lines $t = 0$, $t = \pi$, t-axes, and the graph of the function $f(t) = 2 + \sin t$.

4. Show that the derivative of the function $f(x) = \ln(x + \sqrt{x^2+1})$ is $f'(x) = \frac{1}{\sqrt{x^2+1}}$. Use this fact to determine area enclosed by the graph of $f(t) = \frac{1}{\sqrt{t^2+1}}$, t-axes, and the lines $t = 1$ and $t = 3$.

5. Find the area enclosed by the curves $y = 8/t$, $y = 1/\sqrt{t}$, the t-axis, and the lines $t = 1$ and $t = 8$

6. Show that the derivative of the function $f(t) = \ln(t^2+1)$ is $f'(x) = \frac{2t}{t^2+1}$. Use this fact to determine area enclosed by the graph of $f(t) = \frac{1}{\sqrt{t^2+1}}$, t-axes, and the lines $t = 1$ and $t = 3$. Suppose that the inverse demand function for a good is

$$p^D(q) = \frac{2q}{q^2+1},$$

and equilibrium quantity is 10. Calculate the consumer surplus.